A scaled helix for breakdown studies*

Alex Friedman
LLNL and the Heavy Ion Fusion Science Virtual National Laboratory

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Recent experiments exploring the Pulse-Line Ion Accelerator (PLIA) concept have experienced electrical flashover, and studies are underway to clarify the cause and identify mitigation techniques. Here the possible use of a small, inexpensive helix to explore these phenomena is considered.

The desiderata for such a helix are: smaller size; for a harmonic wave of frequency ω and wavelength λ, product of wave number ($k = 2\pi/\lambda$) and radius (a) of the helical winding unchanged by the scaling; maximum voltage gradient along insulator wall (E) unchanged; trajectories of wall-emitted electrons geometrically unchanged; ratio of electron lifetime τ_e to wave period ($2\pi/\omega$) unchanged; and geometry of electric and magnetic field lines (perhaps in presence of an externally applied solenoid field) unchanged. In this note, it is shown that a suitable scaling exists mathematically, and may well be achievable in practice.

Scaling relations

Here, a superscript tilde ($\sim$) denotes the parameters of the reduced-size helix, so that $\tilde{a}$ is the radius of the scaled helix. The linear dimensions of the original helix are to be multiplied by the factor $1/\alpha^2$ so as to yield the scaled helix, *e*.g.,

$$\tilde{a} = a/\alpha^2. \tag{1}$$

We anticipate that a useful value of α for present purposes will be of order two, leading to a four-fold reduction in the helix radius. The geometry is shown schematically in Fig. 1, which indicates the locations of adjacent maxima and minima of the voltage V on both the original and scaled helices. These extrema are separated axially by distances $L = \pi/k$ and $\tilde{L} = \pi/\tilde{k}$, respectively, and $\tilde{L} = L/\alpha^2$. A typical electric field line is also shown.

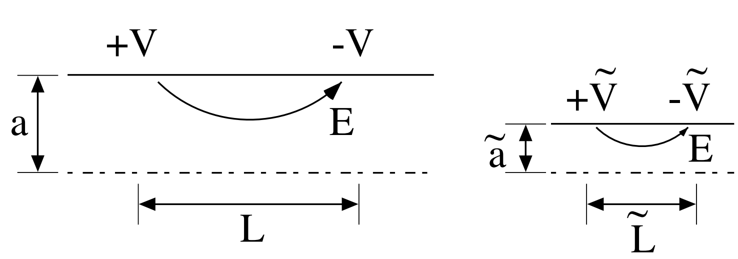

Figure 1: Illustration of original (left) and scaled (right) helices.

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The condition $\tilde{k}\tilde{a} = ka$ yields:

$$\tilde{k} = \alpha^2 k. \tag{2}$$

The "circuit speed" v_c at which a wave travels down a transmission line is given by:

$$v_c = (L_0 C_0)^{-1/2}, \tag{3}$$

where L_0 and C_0 are respectively the inductance and capacitance per unit length. For a non-dispersive line, $v_c = \omega/k$ and so $\tilde{\omega}\tilde{a}/\tilde{v}_c = \omega a/v_c$, giving:

$$\tilde{\omega}/\tilde{v}_c = \alpha^2 \omega/v_c, \tag{4}$$

and (at this point) it might appear possible to alter either ω or v_c alone for a scaled solution; however, as will be seen below, due to other constraints both must be scaled.

The peak magnitude of the electric field must be unchanged; since $E = kV$, $\tilde{k}\tilde{V} = kV$, and so:

$$\tilde{V} = V/\alpha^2. \tag{5}$$

The particle acceleration $\ddot{\mathbf{x}}$ due to the electric field $\mathbf{E}$ is given by $\ddot{\mathbf{x}} = (q/m)\mathbf{E}$, where q and m are respectively the charge and mass of the particle in question (in this case, an electron associated with the observed flashover, but ion dynamics in an actual accelerator would obey the same scaling considered here). Because we want the space and time scales of the particle motion to scale with those of the system and the wave, we require $\ddot{\tilde{\mathbf{x}}}/(\tilde{a}\tilde{\omega}^2) = \ddot{\mathbf{x}}/(a\omega^2)$. Since E is unchanged, $\ddot{\tilde{\mathbf{x}}} = \ddot{\mathbf{x}}$ at a corresponding location in the field pattern, and we see that $\tilde{a}\tilde{\omega}^2 = a\omega^2$, leading to:

$$\tilde{\omega} = \alpha\omega. \tag{6}$$

Correspondingly, the timescales are related by

$$\tilde{\tau} = \tau/\alpha. \tag{7}$$

Similarly, the constancy of $E = kV$ implies that $\tilde{\omega}\tilde{V}/\tilde{v}_c = \omega V/v_c$, and using expressions already developed we see that

$$\tilde{v}_c = v_c/\alpha. \tag{8}$$

Note that the constancy of the peak E and of the shape of the field lines implies that the shapes of the trajectories will be unchanged by the scaling, provided that they are "launched" with zero speed, or with a suitably scaled speed. The characteristic trajectory timescale τ will scale as:

$$\tilde{\tau} = \tau/\alpha. \tag{9}$$

The ratio of magnetic to electric forces must also be unchanged if the trajectory shapes are to be similar. Thus $\tilde{\mathbf{E}}/(\tilde{\mathbf{v}} \times \tilde{\mathbf{B}}) = \mathbf{E}/(\mathbf{v} \times \mathbf{B})$, where v is the instantaneous particle speed. Constancy of E implies that:

$$\tilde{B} = \alpha B, \tag{10}$$

where as usual B is a peak magnitude. The ratio applies to both the helix-generated field and the field due to any external solenoid.

The circuit speed v_c given by Eq. 3 must be scaled; approximate expressions for a helix's inductance and capacitance lend the necessary guidance:

$$L_0 = \pi n^2 a^2 \mu_0 (1 - a^2/b^2) \tag{11}$$

$$C_0 = 2\pi\epsilon / \ln(b/a), \tag{12}$$

where b is the radius of outer pipe wall and n the number of turns of the helix per unit axial length.

The peak magnetic field due to the current in the helical winding is, from ref. [1],

$$B = V/(\pi a^2 n v_c). \tag{13}$$

From Eq. 8, we see that:

$$\tilde{L_0}\tilde{C_0} = \alpha^2 L_0 C_0. \tag{14}$$

From Eqs. 10 and 13, we see:

$$\frac{\tilde{B}}{B} = \frac{\tilde{V}}{V}\frac{a^2 n v_c}{\tilde{a}^2 \tilde{n} \tilde{v}_c} = \alpha, \tag{15}$$

so that:

$$\tilde{n} = \alpha^2 n. \tag{16}$$

Note that the product an and consequently the "pitch" of the helix $(2\pi an)^{-1}$ are preserved by the scaling. That is, the turn-to-turn axial separation scales with the helix radius.

Fixed aspect ratio $\beta \equiv a/b$

Under the additional assumption that the aspect ratio $\beta \equiv a/b$ is unchanged in the transformation to the scaled helix, we obtain:

$$\tilde{n}^2 \tilde{a}^2 \tilde{\epsilon} = \alpha^2 n^2 a^2 \epsilon \quad \text{(fixed } \beta\text{)}. \tag{17}$$

Using Eqs. 1 and 16 in Eq. 17 we obtain the required scaling for the dielectric constant that appears in the helix's capacitance to ground per unit length:

$$\tilde{\epsilon} = \alpha^2 \epsilon \quad \text{(fixed } \beta\text{)}. \tag{18}$$

Thus, if the aspect ratio β is kept unchanged, to realize a change in size it is necessary to employ a different dielectric material (with, for a smaller helix, a higher dielectric constant) in the space between the scaled helix and its outer pipe wall. Since the set of available dielectrics is limited, so are the possible scalings. Relaxing the assumption of constant β opens up a wider range of possibilities, as discussed next.

General aspect ratio β

The assumption of the previous subsection may be relaxed; since the outer pipe wall is not directly involved in the flashover, there is no particular need for β to remain fixed. In this case,

$$\tilde{L_0} = \frac{(1-\tilde{\beta}^2)}{(1-\beta^2)} L_0, \tag{19}$$

$$\tilde{C_0} = \frac{\tilde{\epsilon}\ln\beta}{\epsilon\ln\tilde{\beta}} C_0, \tag{20}$$

and Eq. 18 becomes:

$$F(\tilde{\beta}, \beta, \tilde{\epsilon}/\epsilon) \equiv \sqrt{\frac{\tilde{\epsilon}(1-\tilde{\beta}^2)\ln\beta}{\epsilon(1-\beta^2)\ln\tilde{\beta}}} = \alpha. \tag{21}$$

For a physically realizable solution to exist, this equation must be solvable under the constraint that $\tilde{\beta} < 1$. The necessary combinations of $\tilde{\beta}$ and $\tilde{\epsilon}/\epsilon$ can be arrived at graphically by plotting $F(\tilde{\beta}, \beta, \tilde{\epsilon}/\epsilon)$ and noting where the curve passes through the ordinate value α.

Scaling from the "oil helix"

Moving to a concrete example, we consider the existing "oil helix" at LBNL with $a = 8.1$ cm and $b = 11.5$ cm so that $\beta = 0.689$. A typical value of ϵ for oil is 2.3.

First, we consider the case of an unchanged dielectric constant. Here, the largest α achievable is 1.2, leading to a reduction in scale by a factor 1.44, in the limit as $\tilde{\beta}$ approaches unity. See Fig. 2(a).

Examples: 4x reduction in scale β

If we wish to reduce the linear dimensions by a factor of four, then $\alpha = 2$. In this case, no solution exists for $\tilde{\epsilon}/\epsilon < 2.8$ (approximately), since the curve $F(\tilde{\beta}, 0.689, 2.8)$ passes through the ordinate value 2 at an abscissa value $\tilde{\beta} \simeq 1$; of course, this limiting case is not itself physically realizable. See Fig. 2(b). From this figure it is also possible to read out the value of $\tilde{\beta}$ that would be needed for a more modest reduction in size, for $\tilde{\epsilon}/\epsilon = 2.8$. For example, for $\tilde{\beta} = 0.689$, that is, the original aspect ratio, this higher dielectric constant would allow $\alpha = 1.7$ (approximately), leading to a reduction of the linear dimensions by a factor of about 2.9.

When $\tilde{\epsilon}/\epsilon = 3.0$, a solution with $\alpha = 2$ is obtained at $\tilde{\beta} = 0.935$ (approximately). See Fig. 2(c).

When $\tilde{\epsilon}/\epsilon = 3.5$, a solution with $\alpha = 2$ is obtained at $\tilde{\beta} = 0.8$ (approximately). See Fig. 2(d).

When $\tilde{\epsilon}/\epsilon = 4$, a solution with $\alpha = 2$ is obtained at $\tilde{\beta} = 0.689$, that is, the original aspect ratio, as is evident from Eq. 18. This would require $\tilde{\epsilon} = 9.2$, perhaps achievable with epoxy (at the expense of flexibility) or with glass or other beads in oil.

Other options

It should be noted that, with the above "strict" scaling, the electron velocities are reduced by a factor α relative to their values in the full-sized system. Thus, wall impacts may be less likely to produce secondary electrons. If this proves to be an issue, the operating voltage could be increased above that scaled from the value at which flashover was observed. The experiment would still be in a relevant regime, even with a voltage increased by a factor as large as α (restoring peak energies to of order those in the full-sized system), because flashover was already a problem at voltages well below those desired for PLIA operation. Such a larger voltage would correspond to a regime in which the PLIA would be an attractive accelerator technology.

The question remains of whether a true scaling is too restrictive. It may be the case that the ratio of electric to magnetic forces need not be preserved in order to preserve qualitatively correct dynamics.

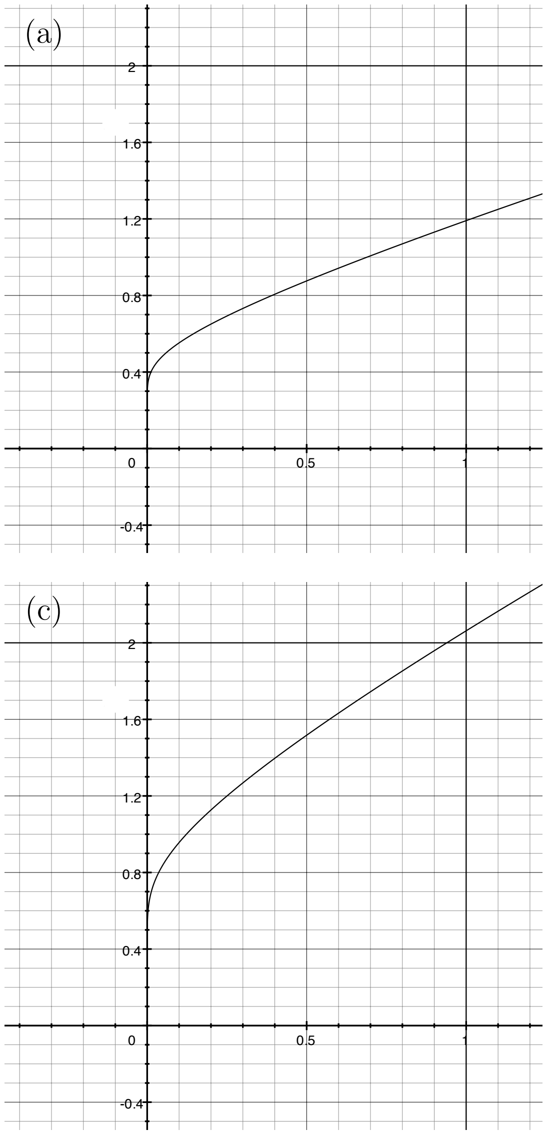

Figure 2: Plots of achievable α as functions of $\tilde{\beta}$ for various values of $\tilde{\epsilon}/\epsilon$): (a) $F(\tilde{\beta}, 0.689, 1.0)$; (b) $F(\tilde{\beta}, 0.689, 2.8)$; (c) $F(\tilde{\beta}, 0.689, 3.0)$; and (d) $F(\tilde{\beta}, 0.689, 3.5)$.

By relaxing the condition of Eq. 15, we may use a more moderate $\tilde{\epsilon}$ along with a larger $\tilde{n}$ than the value specified in Eq. 16, that is, a finer pitch on the helix, to achieve the required wave speed v_c.

WARP tests

To check that the scaling developed here is indeed correct, a WARP run representative of those previously carried out exploring the behavior of test electrons emitted from the insulator surface was repeated with a scale factor $\alpha = 2$. The wave was numerically propagated down the model helix (a transmission line with suitably scaled C_0), and when it had traveled a suitable distance the helix-induced electric and magnetic fields were computed via Poisson equations. Then these fields were used as the basis for a simulation that advanced a number of test particles in the wave frame (they are launched with zero speed in the lab frame, and so appear to be moving backward in the wave frame as they are launched). All the expected scalings were observed, and the orbits are almost all visually identical except for the scaling. One orbit, which lived an unusually long time and is noticeable at lower left because of the dense red trace it left as it moved slowly, differs slightly from that of the corresponding test particle in the reference run; this is attributable to accumulated roundoff differences. A color "contour" plot of the potential in each of the two cases, with test particle trajectories overlaid, is shown in Fig. 3.

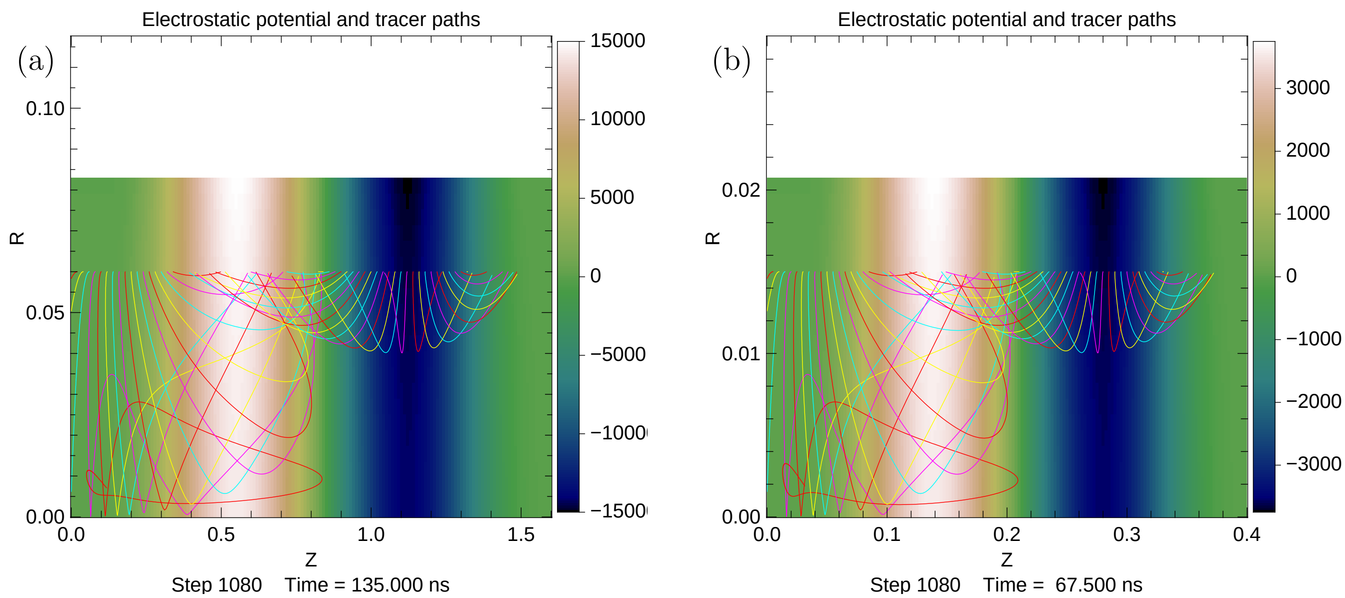

Figure 3: Plots of electrostatic potential with test particle trajectories overlaid for two WARP runs: (a) Reference case, corresponding to an oil helix in which flashover was observed; (b) Scaled case.

Applied magnetic field

We now consider the applied magnetic field that will be needed to test the efficacy of breakdown suppression techniques. WARP studies show that, in the presence of an applied magnetic field, the combined self- and applied fields are nearly parallel to the insulator wall (of radius 6 cm in these runs, as in the oil helix experiment), but "dip" below it. Furthermore, strongly magnetized electrons exhibit orbits that are bound tightly to the lines of $\mathbf{B}$. Examples are shown in Fig. 4.

In these runs the oil helix was modeled. The peak magnetic self-field magnitude is 0.002 T; a peak applied voltage of 15 kV was used. In Fig. 4(a) a trajectory and a nearby field line from run 501, with a 0.2-T applied field, are shown. For this run the field line, which in the absence of the helix field would be nearly overlaid with the insulator wall, dips by d =0.30 mm. The gyromotion appears as small "wiggles" superposed on the trajectory trace. Fig. 4(b) shows a corresponding field line and multiple orbits from run 502 with a 1-T applied field, for which d =0.06 mm; relative to the previous run, d is reduced by the expected ratio.

As a check, in Ref. [2] R. Briggs lists for the oil helix $\pi a^2 n v_c = 9.5 \times 10^6$ V/T, leading to a radially-averaged magnetic field from Eq. 13 of 0.0016 T = 16 G, somewhat down from the peak in the WARP runs but basically consistent.

It may be important that grading rings intercept those secondary electrons emitted from the insulator surface that would otherwise lead to flashover. It is thus desirable that d be smaller than

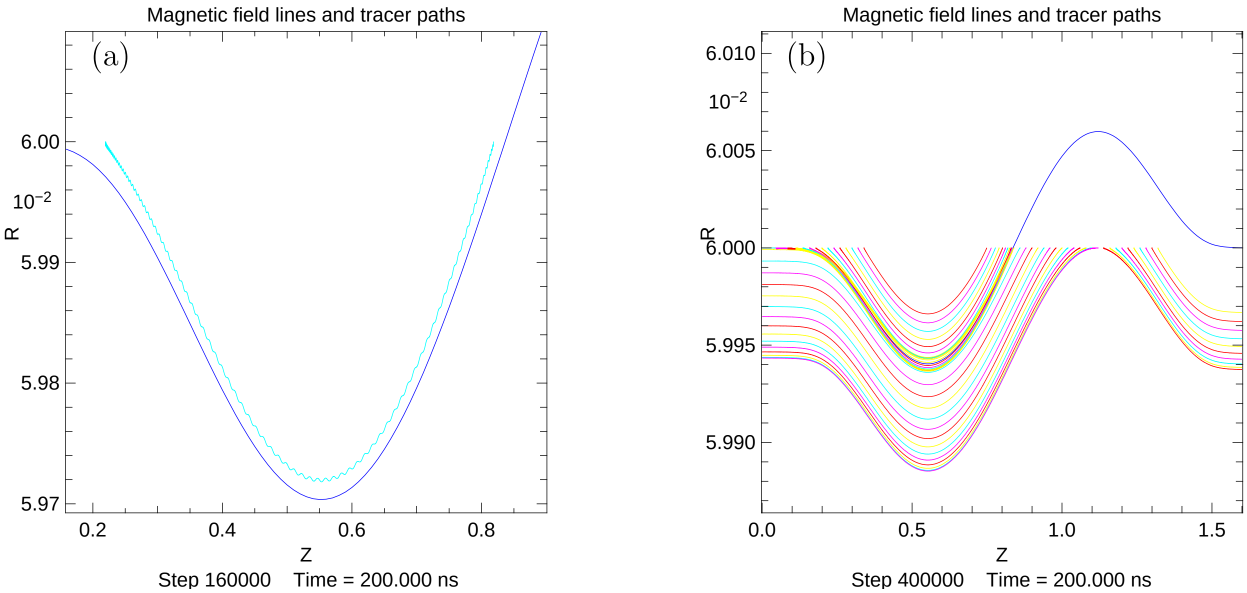

Figure 4: Plots of magnetic field lines and test particle trajectories for two WARP runs including applied solenoid fields: (a) Run 501, with a 0.2-T applied field; (b) Run 502, with a 1-T applied field. Note expanded vertical scales; $\mathbf{B}$ is nearly parallel to the insulator surface.

the distance by which a grading ring extends radially inward from the insulator surface. Since such rings would probably extend at least 3 mm, the existing oil helix would require, for this pulse, a field 1/10 as large as that used in run 501, or 0.02 T. However, the ± 15 kV voltage peak used is much smaller than desired, and operation at (say) ± 150 kV would require a 0.2 T solenoid field for d to be small enough. Grading rings extending 12 mm would require a 0.05 T applied field.

For a scaled helix with dimensions reduced by a factor of four, the scaling developed here suggests that the applied field should be increased by a factor $\alpha = 2$, leading to a solenoid that produces 0.4 T to keep d small enough (for grading rings scaled down to 0.75 mm). For thicker rings extending 3 mm, a 0.1 T applied field would suffice.

Scaled systems based on glass tubing (inner radius 0.911 inch)

We now consider example scaled systems based on readily available glass tubing and dielectric materials. The tubing is of length 17.75 in = 45.1 cm between flanges, and of 2 inch outer diameter (O.D.); the outer radius is $r_o = 2.54$ cm = 1 in. The inner diameter (I.D.) of the tubing is 1-13/16 in (inner radius $r_i = 2.301875$ cm = 0.90625 in). Scaling from the existing helix with insulator wall radius r_i =6 cm and $a = 8.1$ cm would yield a nominal scaled helix radius of 3.108 cm, but (if r_i/a and $\beta = a/b$ are both held constant) a dielectric with the high constant corresponding to such a scaling may be impractical. Instead, a mixture of oil and beads of soda-lime glass has $\tilde{\epsilon} = 5$, and a scaled system based on this dielectric would have $\alpha = \sqrt{5/2.3} = 1.474$. There are two limiting cases with this dielectric, summarized in Table 1.

With constant $\beta = a/b$, the scaled helix would have $\tilde{a} = 3.726$ cm = 1.467 in, and (using $\tilde{\beta} = \beta = 0.689$) a pipe wall inner radius of $\tilde{b} = 5.29$ cm = 2.083 in. In this case, the scaled "thin" grading rings would extend 1.38 mm and the required solenoid field would be 0.295 T; the scaled "thick" grading rings would extend 5.52 mm and the required solenoid field would be 0.074 T. The imperfection in this scaling is that the insulator radius is quite small relative to the helix radius, so that electron orbits would launch from, and be absorbed in, a field that differed geometrically from that in the full-scale helix.

Alternatively we may use, with the same 1-inch radius glass pipe, the nominal $\tilde{a} = 3.108$ cm $= 1.223$ in and $\alpha = 1.615$ with $\tilde{\epsilon} = 5$. In that case, instead of the above pipe radius corresponding to $\tilde{\beta} = \beta$, we would use a somewhat smaller pipe radius. Solving, we find $\tilde{\beta} = 0.8419$ and so $\tilde{b} = 3.692$ cm $= 1.453$ in. In this case, the scaled "thin" grading rings would extend 1.11 mm and the required solenoid field would be 0.323 T; the scaled "thick" grading rings would extend 4.45 mm and the required solenoid field would be 0.081 T. The spacing between the insulator outer radius (2.54 cm) and the helix mean radius ($\tilde{a}$) is 0.57 cm, and the spacing between the helix mean radius and the pipe inner radius ($\tilde{b}$) is 0.58 cm.

	Configuration		
	Full-sized oil helix	Scaled with constant $\beta = a/b$	Scaled with constant r_i/a
r_i (insulator inner radius)	6.00 cm = 2.36 in	2.30 cm = 0.91 in	2.30 cm = 0.91 in
a (helix radius)	8.10 cm = 3.19 in	3.73 cm = 1.47 in	3.11 cm = 1.22 in
b (pipe inner radius)	11.5 cm = 4.53 in	5.29 cm = 2.08 in	3.69 cm = 1.45 in
$a - r_o$	1.8 cm (est.)	1.19 cm	0.57 cm
$b - a$	3.4 cm	1.56 cm	0.58 cm
α^2 (helix radius factor)	1.0	2.17	2.61
β (aspect ratio a/b)	0.689	0.689	0.842
ϵ (dielectric constant)	2.3	5.0	5.0
"Thin" grading ring	3 mm	1.4 mm	1.1 mm
Solenoid field	0.2 T	0.3 T	0.3 T
"Thick" grading ring	12 mm	5.5 mm	4.4 mm
Solenoid field	0.05 T	0.07 T	0.08 T

Table 1: Parameters for oil helix and two scaled options (using actual I.D. of glass tubing).

Of these two options, the latter (constant r_i/a) appears more attractive, since it more faithfully reproduces the field and trajectory geometry of the oil helix, and is also the more compact of the two options. This scaling is not extreme; in fact, with suitable choice of parameters it appears possible to design the scaled helix with a wave speed to match the speed of the ion beam that can be produced on the NDCX apparatus at LBNL. The oil helix was run with two different wave speeds achieved by varying n. The "fast" oil helix used $n = 159$ turns/m and achieved a wave speed of $v_c = 2.9 \times 10^6$ m/s, while the "slow" helix" had $n = 243$ and $v_c = 1.9 \times 10^6$ m/s; both suffered from flashover. Similarly for the scaled helix, there is freedom to choose n to match the beam speed. If we were to scale from the "fast" oil helix, the wave speed of the resulting small helix with $\tilde{n} = \alpha^2 n_{\text{fast}}$ would be related to that of the "slow" oil helix in the ratio $v_{\text{scaled}}/v_{\text{slow}} = n_{\text{slow}}/n_{\text{fast}}\, \alpha^{-1} = 1.53/1.62 = 0.95$, and could be used in an ion beam test.

Also, if we were to build a small helix with constant r_i/a and a winding spacing scaled from the "slow" oil helix but use pure oil as the dielectric, the wave speed would be related to that of the "slow" oil helix in the ratio $v_{\text{scaled}}/v_{\text{slow}} = \alpha^{-1}\sqrt{\epsilon_{\text{oil+glass}}/\epsilon_{\text{oil}}} = 1/1.62\sqrt{5/2.3} = 0.91$. Decreasing n would allow the wave speed to match that of the "slow" helix. Of course, in contrast with the scenario of the previous paragraph, these are not true scaled solutions; but they may represent attractive practical options.

Acknowledgments

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References

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[2] R. J. Briggs, "Helix fields - some useful relations and parameters" (unpublished note), May 10, 2006.